


**Research paper****Effect of Rayleigh Range on the Accuracy and Measurement Range of Refractive Index for Brix Determination**Ali Mohseniafkham, Meysam Valipourhafshejani, Seyed Hassan Tavassoli**Laser and Plasma Research Institute, Shahid Beheshti University, Tehran, Iran**h-tavassoli@sbu.ac.ir**Article info:****Article history:**

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Abstract

Optical techniques have been widely employed in various sensing and analytical applications in recent years. In this regard, considerable efforts have been devoted to the development and optimization of refractometry-based methods for Brix measurement. In this study, the proposed system was modeled and simulated, with primary emphasis mainly placed on optimizing the angular distribution of the rays via careful adjustment of the Rayleigh range and focal length. The results clearly indicate that this parameter is an important factor in the proper formation of the critical angle and in improving overall system performance. Also, it was found that the selection and positioning of optical components, including the light source, prism, lenses and detector, along with the adjustment of this parameter, have a considerable effect on the quality of the bright–dark boundary and the measurement accuracy. Accordingly, simultaneous optimization of these factors is necessary to achieve optimal performance of the refractometry system.

1. Introduction

Industries such as sugar production, distillate products, honey, maple syrup, and corn syrup mainly deal with solutions in which sugars (including sucrose, glucose, and fructose) constitute the primary solutes. Therefore, accurate determination of sugar concentration in such solutions is of high importance, and this quantity is typically reported in terms of the Brix scale as a standard index [1]. The Brix scale indicates the

amount of dry matter present in solutions that are mainly composed of sucrose. The Brix scale has been extensively used in the sugar industry for many years. Accordingly, sugar chemists define Brix as the weight percentage of sucrose in a sugar solution [2].

Brix is mainly measured using density-based methods such as the hydrometer, Westphal balance, and pycnometer. Among these methods, the hydrometer is considered the most common approach [3]. However, these measurement



methods do not provide sufficient accuracy and efficiency due to operator errors and the manual nature of the process. In contrast, optical techniques have been developed for more precise sensing and analysis. Refractive index is a fundamental and important characteristics of substance [4], [5] There are various methods for measuring the refractive index, including the Michelson interferometer [6], [7], Fabry–Pérot interferometer [7], [8], [9] and the Abbe refractometer [10]. Among these methods, critical-angle-based refractive index measurement is more suitable, as it is not affected by the color of the liquid sample or the suspended particles present in it [11]. A refractometer optically measures the density of a liquid and is then calibrated in terms of refractive index. This device is also typically equipped with a scale in degrees Brix. A refractometer is suitable as a tool for measuring solution concentration in chemical, food, and beverage industries, and it is extensively used for quality control testing [12]. Although the principle of critical angle based refractometers is well established, optimization of various structural parameters including the source position, prism, detector, source divergence angle, prism refractive index, prism size, angular position and height of the source relative to the prism surface, and the detector position for optimal reception of reflected light collectively determines the sensitivity and performance of the refractometer [13]. In this study, an Abbe refractometer was designed and implemented using an LED light source at a wavelength of 589 nm (close to the sodium D-line as a standard reference in refractometry) and a BK7 prism, such that its operating range is consistent with the Brix variation range. To achieve total internal reflection conditions within this range, optical parameters including the incidence angle and Rayleigh range were optimized through simulation in Zemax software to provide an appropriate angular distribution for the formation of the critical angle at the prism–

sample interface. The obtained results indicate that the optimized design, while maintaining the desired range, enables discrimination of close Brix values with a minimum difference of 0.18 degrees.

2. Materials and methods

The operation of the Abbe refractometer is based on Snell's law of refraction. In this system, a convergent cone of light rays from a light source is directed toward the prism and the sample. A portion of the rays undergoes total internal reflection at the prism–sample interface, while the remaining portion is transmitted through the sample and exits into air. The reflected rays form an image in which the boundary between the bright and dark regions is determined by the critical angle of total internal reflection and, consequently, depends on the refractive index of the solution. The operating principle of the refractometer is shown in Figure 1. In a refractometric system, a relationship is established between the incident angle of light, the refractive index of the prism, and the refractive index of the sample [14], such that the incident angle directly influences the conditions for total internal reflection and the formation of the critical angle, thereby providing the basis for determining the sample's refractive index.

Eq. (1) expresses the relationship between the refractive index and the angle of incidence in the first region and the refractive index and the angle of refraction in the second region.

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad (1)$$

It is clear that the angle of refraction cannot exceed 90° . By substituting $\theta_2 = 90^\circ$ and $\theta_1 = \theta_c$ into Eq. (1), the expression for the critical angle is obtained.

$$\theta_c = \arcsin \frac{n_2}{n_1} \quad (2)$$

According to NIST standards, a sample with zero Brix (distilled water) has a refractive index of

1.3333, while a sample with 80 Brix has a refractive index of 1.4614. Considering the use of a BK7 prism with a refractive index of 1.517, the critical angle at the prism–solution interface is calculated to be 61.510° for the zero-Brix sample and 74.439° for the 80-Brix sample. It should be noted that the boundary between the bright and dark regions does not form as a sharp step-like transition, but rather appears as a gradual change. This phenomenon is mainly attributed to the spectral bandwidth (FWHM) of the light source. Since the refractive index of the prism is wavelength-dependent, each spectral component produces a different critical angle, resulting in a broadened bright–dark boundary on the detector. In addition, the optical design of the system and the position of the detector also influence the narrowing or broadening of this transition region. A reduction in the width of the bright–dark boundary leads to a more precise determination of

the critical angle and, consequently, improves the system accuracy in resolving closely spaced Brix values. Therefore, the design should be carried out in a way that the incident rays generate the required angular range at the prism–sample interface, such that for a zero-Brix sample the bright–dark boundary appears at the end of the detector array, and for an 80-Brix sample it appears at the beginning, ensuring that the entire pixel array is utilized in the measurement process. This condition is achieved through adjustment of the incident angle and the Rayleigh distance, as these two parameters define the angular distribution of the rays in the boundary region. In addition, the ray path must be controlled so that propagation occurs only within the prism–sample contact region, since interaction with the prism–air interface leads to unwanted total internal reflection over a wide angular range, which in turn distorts the measurement and degrades accuracy.

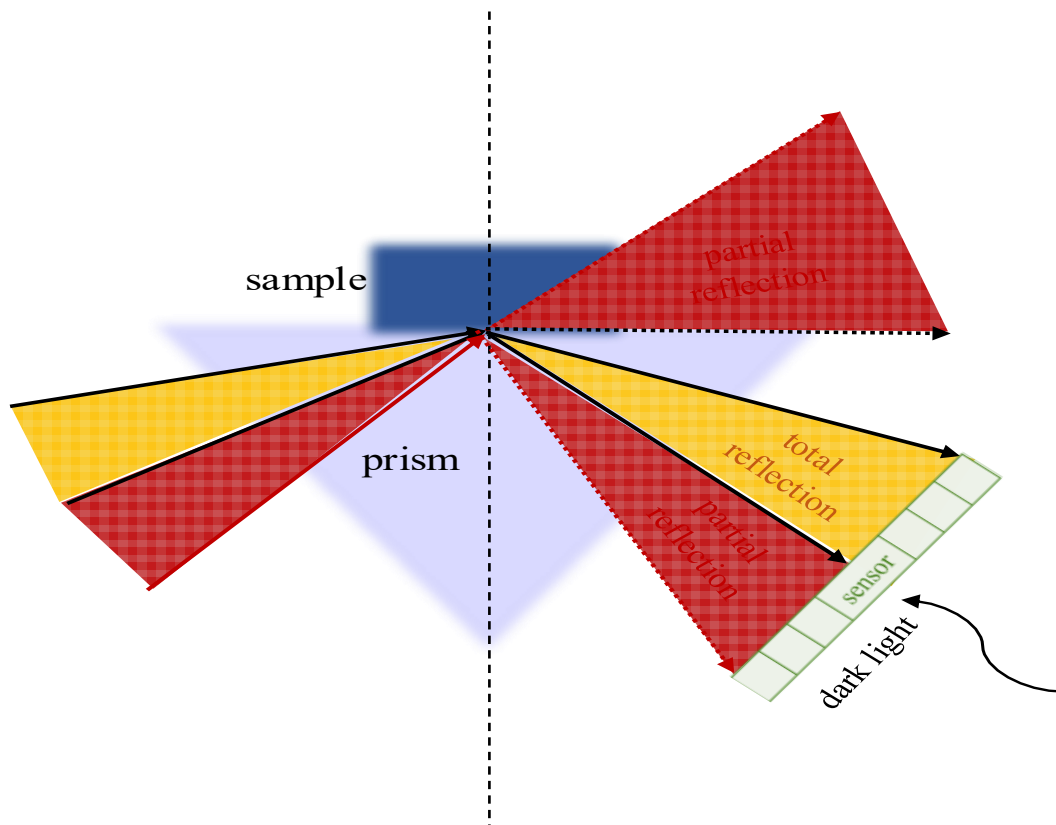


Figure 1. Schematic of refractometric principle based on total internal reflection and critical angle at the prism–sample interface

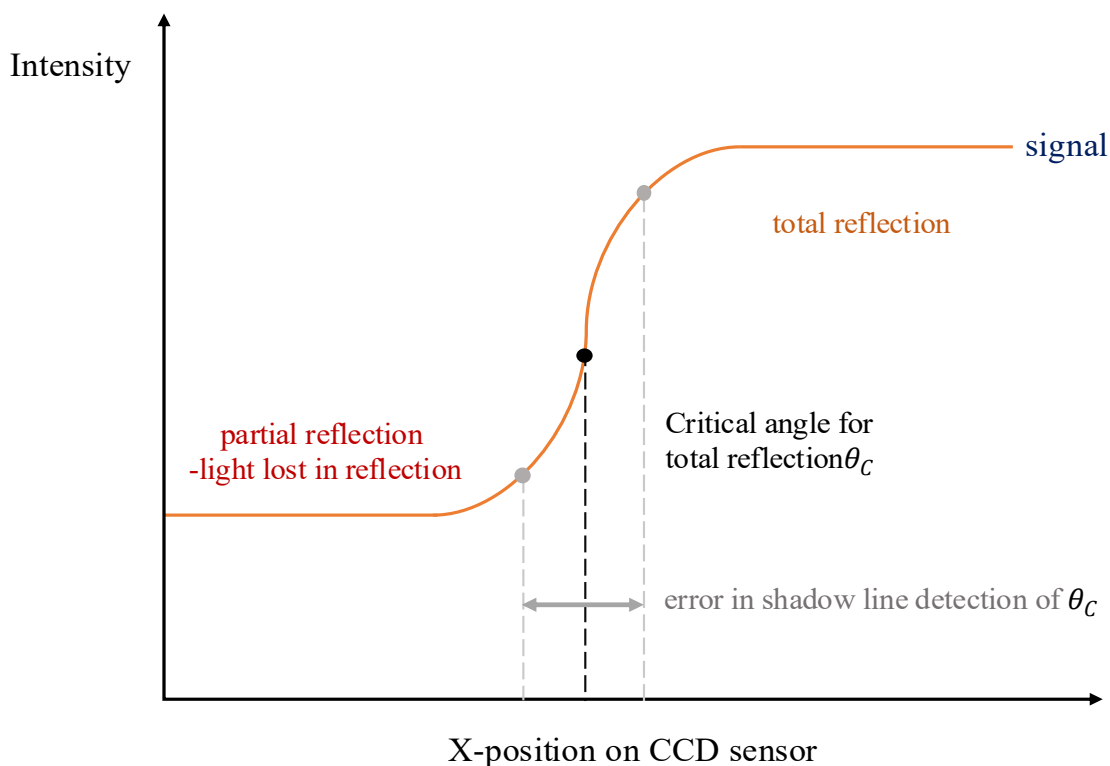


Figure 2. The bright–dark boundary observed on the detector appears as a gradual transition region; the center of this region corresponds to the critical point, while its width reflects the uncertainty in determining the exact position of the critical point and can contribute to measurement error

3. Results and Discussion

Different configurations for the arrangement of optical components, including the light source, prism, and detector, were evaluated. Simulation results obtained from Zemax software indicate that adjustment of the Rayleigh range plays a significant role in controlling the angular distribution of the rays at the prism–sample interface. In general, the critical angle varies with changes in the sugar content of the solution, while factors such as color, gas, and dissolved particles do not affect the measurement result. Using this model, the Brix value of the sample can be obtained directly. Since total internal reflection occurs only within a specific angular range, improper selection of the Rayleigh range may cause a portion of the rays to reach the prism–sample interface outside the desired angular range.

Under such conditions, the bright–dark boundary is not properly established, and multiple separate boundaries may even appear on the detector, leading to the extraction of incorrect data.

To investigate this effect, different configurations with various Rayleigh ranges were simulated in Zemax software, and the corresponding results are showed in [Figure 3](#). In the optimized configuration, the angular distribution of the rays is adjusted such that only a single bright–dark boundary appears within the detector range, whereas in non-optimized designs, variation of the Rayleigh range results in undesired reflections and the appearance of multiple transition regions on the detector. The system performance was evaluated using a one-dimensional Toshiba TCD1304AP array detector with 3648 pixels, and the bright–dark boundary was extracted. To increase the accuracy of critical point

determination, a smoothing process was applied. Experimental results showed that for samples with Brix values of 30 and 32, the critical point was located at pixels 847 and 901, respectively.

Furthermore, the maximum intensity variation in the transition region between darkness and brightness was approximately 10 pixels, which is considered the effective width of the boundary.

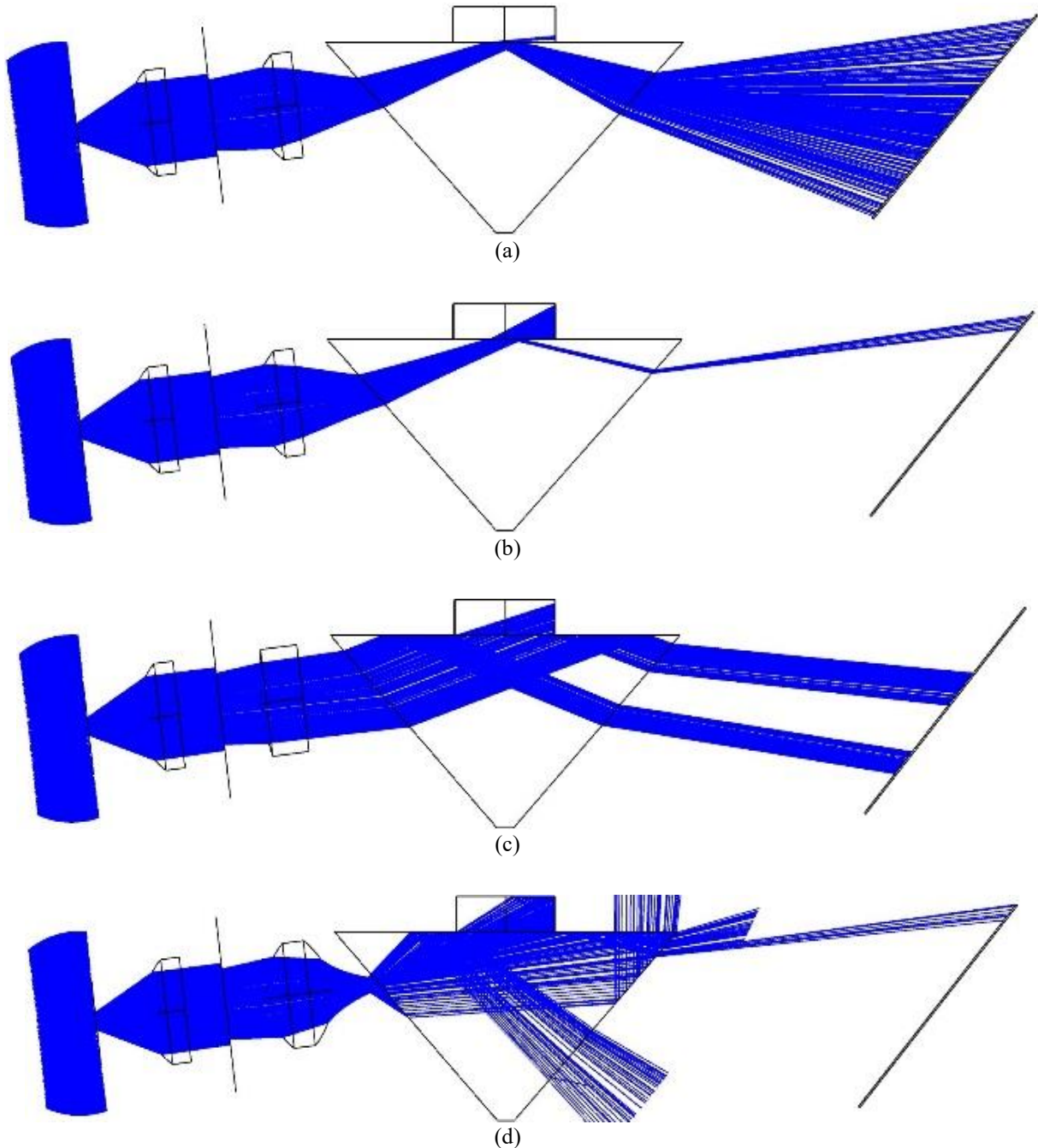


Figure 3. Comparison of system performance under different Rayleigh range configurations. In Figure 3(a) and Figure 3(b), appropriate selection of the Rayleigh range leads to the proper formation of the bright–dark boundary and discrimination between two samples with different refractive indices. In contrast, Figure 3(c) and Figure 3(d) related to non-optimized configurations with inappropriate Rayleigh ranges. In Figure 3(c), despite identical refractive indices of the samples, multiple bright–dark boundaries appear on the detector, while in Figure 3(d), undesired internal reflections within the prism lead to the generation of incorrect data

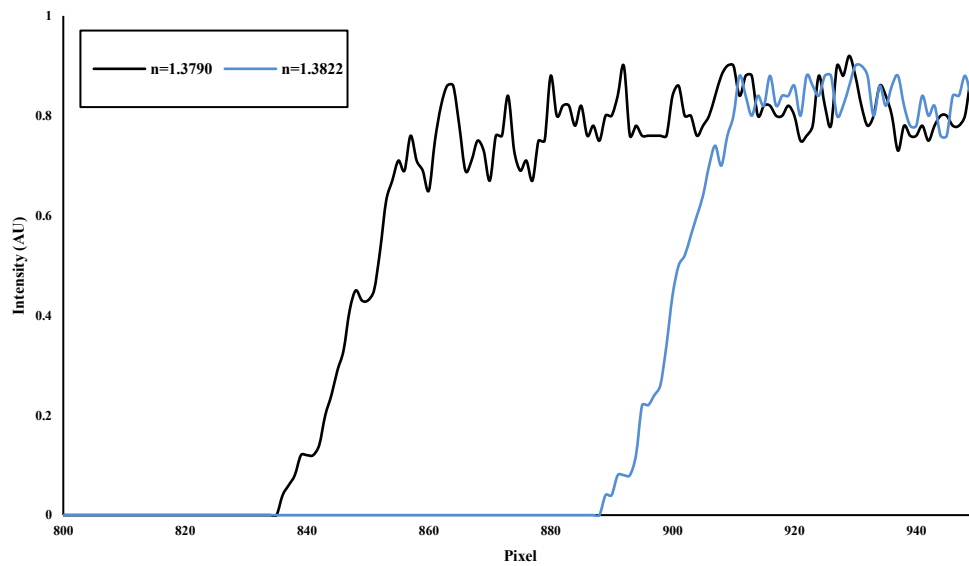


Figure 4. The intensity variations and displacement of the bright–dark boundary as a function of detector pixels are illustrated for two samples with different refractive indices

For system calibration and coverage of the desired refractive index range, four reference materials including distilled water, ethanol, cyclohexane, and chloroform were used. By obtaining the position of the bright–dark boundary for each sample and fitting the data, a relationship between the pixel position and refractive index was obtained, which was then used for refractive index determination and Brix measurement of the samples.

Table 1. Comparison of standard and measured Brix values along with the corresponding measurement errors. This table demonstrates the accuracy of the system at different Brix levels by presenting the relative error with respect to the reference values

Sample (Brix)	Standard Refractive	Measurement Refractive	Relative Error (%)
6	1.3417	1.3411	0.04474
12	1.3509	1.3488	0.1557
18	1.3602	1.3586	0.1178
24	1.3696	1.3671	0.1829
30	1.3790	1.3783	0.05079
32	1.3822	1.3798	0.1739
36	1.3886	1.3845	0.2961
42	1.3982	1.3956	0.18630

Using this Results in the designed system, a displacement of approximately 0.037 Brix per pixel and a resolution of about 0.18 Brix were estimated. These results indicate the capability of the system to discriminate small Brix variations within the investigated range. Table 1 shows the measurement results obtained at a wavelength of 589 nm and a temperature of 25°C.

4. Conclusion

A total internal reflection-based Abbe refractometer was designed, simulated, and implemented using a BK7 prism and an LED light source at a wavelength of 589 nm. In the system design, the effect of the Rayleigh range on the angular distribution of the rays and the formation of the critical angle was investigated, and simulation results in Zemax software showed that appropriate selection of this parameter plays a decisive role in the proper formation of the bright–dark boundary and system performance. It was also found that improper adjustment of the Rayleigh range can lead to the formation of multiple bright–dark boundaries and the

extraction of erroneous data. The designed system was calibrated using reference materials, and its performance was experimentally evaluated over a Brix range of 0 to 80. The results demonstrated that the system is capable of measuring Brix variations with a resolution of approximately 0.18 °Brix. In addition, the use of simple and readily available optical components enables the implementation of a low-cost system suitable for refractometric applications, particularly in the food industry.

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